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On the Interpolation of Logarithmic Series. By JAMES MEIKLE, *Assistant Actuary of the Scottish Provident Institution.*

[Read before the Institute, 31st March, 1856, and a notice of it ordered by the Council to be printed.]

IN this paper Mr. Meikle states it to be his object to simplify and popularize what has been written by Mr. Gompertz and Mr. Farren on the subject in question. He says, "In the construction of tables of mortality, the results arising out of the statistics, viz. the logarithms of the number alive at the beginning of each year, or the logarithms of the probability of living one year, generally proceed so irregularly from age to age that it is deemed expedient to graduate and reduce them to a more consistent and harmonious form. For this purpose it is thought that the characteristic features of the table which is being developed will be as equally retained in any set of equidistant terms of the series as they are now exhibited in the rough and irregular items of the table. Selecting, therefore, the terms of the series at ages 10, 20, 30, &c., or at ages 10, 20, 40, &c., and making these the fixed points of a new series, it is required to interpolate for the values at the intermediate ages."

Mr. Meikle proceeds to describe Mr. Gompertz's method, which, he says, "as applied to the interpolation of series, resulted more from a search after some general law, thought to be inherent in tables of mortality, than from an investigation into the methods of graduation or interpolation. Mr. Gompertz observed, that out of a given number alive at one age, the number which according to the original tables survived each succeeding age differed in a very trifling degree from that which he brought out according to his hypothesis; viz., that the law of human mortality proceeded in a geometrical ratio." Mr. Meikle then shows in what manner and to what extent the numbers living, according to the Carlisle Table, at the ages between 10 and 20, are modified by this hypothesis, and continues:—

"The above may suffice to show—but if it had been continued a little further it would have been more obvious from the progressive differences—that the second table was preferable to the first, notwithstanding the circumstance of its being based upon a mere hypothesis; and that, sufficiently exact for all practical purposes, it retained the characteristic features of the original statistics."

The writer proceeds to give one of the methods by which Mr. Gompertz arrives at his formula, which, as that gentleman's treatise is not generally accessible, we insert briefly as follows :—

Let

λy_a =log. of number living at youngest age in the original table
λy_{a+r} = " " r years older
λy_{a+2r} = " " $2r$ "
&c. &c. &c.;

and let

$$\begin{aligned}\lambda y_a - \lambda y_{a+r} &= m\\ \lambda y_{a+r} - \lambda y_{a+2r} &= mp\\ \lambda y_{a+2r} - \lambda y_{a+3r} &= mp^2\\ \&c. \quad \&c.\end{aligned}$$

By addition,

$$\lambda y_{a+n-r} - \lambda y_{a+n} = mp^{\frac{n}{r}-1},$$

and

$$\lambda y_a - \lambda y_{a+n} = m + mp + mp^2 + \&c.\ mp^{\frac{n}{r}-1} = \frac{m}{1-p}\left(1-p^{\frac{n}{r}}\right);$$

or, if p be greater than $1 = \frac{m}{p-1}\left(p^{\frac{n}{r}}-1\right)$,

$$\therefore\ \lambda y_{a+n} = \lambda y_a - \frac{m}{1-p}\cdot\left(1-p^{\frac{n}{r}}\right);$$

and if for $\frac{m}{1-p}$ we write λe, we have

$$\lambda y_{a+n} = \lambda y_a - \lambda e + \lambda e p^{\frac{n}{r}} = \lambda\frac{y_a}{e} + \lambda e p^{\frac{n}{r}} = \lambda\frac{y_a}{e}\cdot e^{p^{\frac{n}{r}}};$$

$$\therefore\ y_{a+n} = \frac{y_a}{e}\cdot e^{p^{\frac{n}{r}}}.$$

And writing x for $a+n$, and q for $p^{\frac{1}{r}}$,

$$y_x = \frac{y_a}{e}\cdot e^{q^{-a}\cdot q^x};$$

or substituting d for $\frac{y_a}{e}$, and g for $e^{q^{-a}}$,

$$y_x = d\cdot g^{q^x}.$$

For a practical formula we have, as before,

$$\lambda y_{a+n} = \lambda y_a - \frac{m}{1-p} + \frac{m}{1-p}p^{\frac{n}{r}},$$

or

$$\lambda y_{a+n} = \lambda y_a - \frac{m}{1-p} + \text{N},\ *$$

* A more convenient form of this expression will be found at page 206, vol. IV., of this *Journal.*—Ed. *A. M.*

where

$$N = \frac{m}{1-p} p^{\frac{n}{r}}, \text{ and } \lambda N = \lambda \frac{m}{1-p} + n\lambda \frac{p}{r};$$

or if p be greater than 1,

$$\lambda . y_{a+n} = \lambda y_a + \frac{m}{p-1} - N, \text{ where } \lambda N = \lambda \frac{m}{p-1} + n\lambda \frac{p}{r}.$$

By way of illustrating the use of this formula, Mr. Meikle shows how by its means Mr. Jellicoe's Table of Indian Rates (*see* vol. i., page 169, of this *Journal*) is constructed, and concludes this part of his paper by observing that the other modifications proposed by Mr. Gompertz are not generally adopted because the values obtained by them will not, in any instance, correspond with those of the original table, and that the series can be continued only by a repetition of the process and the use of fresh constants, in which case there will still be a slight break in the harmony of it. Mr. Galloway (says Mr. Meikle) admits that he was obliged "to vary the numbers before and after the middle age, in order to maintain the regularity of the progression in passing from the first formula to the second."

The author then proceeds to describe the method of differences, citing Mr. Farren's use of it in his *Life Contingency Tables*, and suggesting certain modifications of his own, for information as to which the reader is referred to Mr. Meikle's manuscript.

A Problem in Fire Insurance—To apportion a given Loss on Property insured by Specific Policies. *By* THOMAS MILLER, *Esq., of the Scottish Union Insurance Society, London.*

A SPECIFIC POLICY is one which insures stated sums on particular risks, and guarantees the assured against any loss thereon within the limits of the amounts insured. It differs from an AVERAGE OR FLOATING POLICY in several respects—the latter, amongst other privileges, having only to bear a share of any loss compared with the whole loss, similar to the proportion which the amount it insures bears to the whole value of the property insured at the time of the fire.

In solving this problem it requires to be noticed that, by the conditions of insurance, when any stated risk is insured by more